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ON THE INVESTIGATION OF THREE-DIMENSIONAL FLOWS OF PERFECT GAS IN "NARROW" DUCTS*

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A system of integral and differential equations and relations is obtained for strong discontinuities by averaging with respect to variable y of a cylindrical system of coordinates $xy\varphi$. The system defines steady and unsteady three-dimensional flows of perfect (inviscid and non-heat-conducting) gas in ducts of a class that is important in applications. Two walls of such ducts Σ^+ and Σ^- are close to surfaces of revolution and the distance between them is equal to the remainder $y^+ - y^- \ll (y^+ + y^-)/2$, where $y = y^\pm(x, \varphi)$ are equations that define $\Sigma^\pm$. The ensuing reduction of the number of independent variables makes possible mobile numerical simulation, as will be illustrated on examples in which linearized and complete (nonlinear) variants of the obtained system are used. The method of derivation of approximate equations is similar to that used for obtaining two-dimensional equations for the "variable height" layer in turbine blading flows. This obviously results in the congruence of the respective equations.

1. Let us consider a narrow duct of the type shown in Fig.1, where the orientation of normals to $\Sigma^\pm$ is almost everywhere close to the direction of the y-axis. The last constraint together with the stipulated narrowness of the duct means that

$$h \equiv y^+ - y^- \ll Y \equiv \frac{y^+ + y^-}{2}, \quad |y_x^\pm| = \left|\frac{\partial y^\pm}{\partial x}\right| \ll 1, \quad \frac{y_\varphi^\pm}{y^\pm} \equiv \frac{1}{y^\pm}\left|\frac{\partial y^\pm}{\partial \varphi}\right| \ll 1 \tag{1.1}$$

The words "almost everywhere" relate to the second and third of these conditions which may be violated in the small neighborhood g_w of the intersection line (junction) of Σ^+ and Σ^-. The smallness of that neighborhood ensures the closeness of $\Sigma^\pm$ to a surface of revolution, although only with respect to y but not to y_φ. For long ducts the second of conditions (1.1) does not imply smallness of variations of $y^\pm$ and h along the whole (or a considerable part) of the duct length. In addition to the above conditions we introduce a constraint on the shape of the strong discontinuity surfaces. We confine the analysis to flows for which

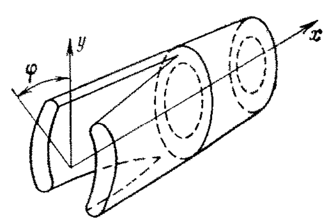


Fig.1

$$D_y^{\circ 2} \ll D_x^{\circ 2} + y^{-2} D_\varphi^{\circ 2} \tag{1.2}$$

where $D^\circ(t, x, \varphi, y) = 0$ is the equation of the discontinuity surface, t is the time, and the subscripts denote respective partial derivatives.

It is shown below that by virtue of the condition of $\Sigma^\pm$ impermeability and of inequalities (1.1) and (1.2), the inequality $v^2 \ll u^2 + w^2$, in which u, v and w are the x-, y- and φ-components of the velocity vector $\mathbf{Q}$, is satisfied everywhere, except possibly in the small neighborhoods g_w. This enables us to determine thermodynamic parameters (pressure p, density ρ, specific entropy s, internal energy e, enthalpy $i = e + p/\rho$, etc.), as well as components u and w independently of v. It is convenient to use the integral laws of conservation of the mass of streams, x-components of momentum, and of the moment of momentum and of energy, as the basis for the derivation of respective equations. These laws are of the form

$$\frac{d}{dt}\iiint\limits_\Omega \rho\, d\Omega + \iint\limits_\sigma \rho Q_n\, d\sigma = 0, \quad \frac{d}{dt}\iiint\limits_\Omega \rho u\, d\Omega + \iint\limits_\sigma \rho u Q_n\, d\sigma + \iint\limits_\sigma p n_x\, d\sigma = 0 \tag{1.3}$$

$$\frac{d}{dt}\iiint\limits_\Omega y\rho w\, d\Omega + \iint\limits_\sigma y\rho w Q_n\, d\sigma + \iint\limits_\sigma y p n_\varphi\, d\sigma = 0, \quad \frac{d}{dt}\iiint\limits_\Omega \rho(2e + Q^2)\, d\Omega + \iint\limits_\sigma \rho(2i + Q^2) Q_n\, d\sigma = 0$$

where Ω is an arbitrary fixed volume bounded by the closed surface σ, free of any bodies inside it, $\mathbf{n}$ is the unit vector of the normal to σ with components n_x, n_y, and n_φ, $Q_n = \mathbf{Q}\cdot\mathbf{n}$ is

*Prikl.Matem.Mekhan.,44,No.4,667-676,1980 the normal to σ projection of $\mathbf{Q}$ and $Q = |\mathbf{Q}|$. As σ we take the surface formed by the duct walls $\Sigma^{\pm}$ and the "side" surface σ°. We orient σ° so that $n_y = 0$ on σ°, although otherwise its position is arbitrary. We introduce besides the true value of any parameter Φ, its mean value

$$\langle\Phi\rangle = \frac{1}{h}\int_{y^-}^{y^+} \Phi\, dy \tag{1.4}$$

In particular $\langle y\rangle = Y \equiv (y^+ + y^-)/2$. By definition $\Phi = \langle\Phi\rangle + \delta\Phi(t, x, \varphi, y)$ with $\langle\Phi\rangle$ independent of y and the integral of $\delta\Phi$ over the duct height is zero. For ducts and flows that satisfy conditions (1.1) and (1.2) we have $\delta\Phi \ll \langle\Phi\rangle$. Because of this the formulas

$$\langle\Psi(\Phi_1, \ldots, \Phi_n)\rangle = \Psi(\langle\Phi_1\rangle, \ldots, \langle\Phi_n\rangle) \tag{1.5}$$

are valid within an error of the order of $(\delta\Phi)^2$ and $h\delta\Phi$.

We carry out integration in (1.3) with respect to y over the volume bounded by the surface σ defined above. We take into consideration the condition of $\Sigma^{\pm}$ impermeability, inequalities (1.1), and formulas (1.4) and (1.5), and neglect in the obtained equations v which is small in comparison with $q = \sqrt{u^2 + w^2}$. If we then omit the averaging symbol and substitute y for Y, since the former is not henceforth used in its original sens, Eqs. (1.3) assume the form

$$\frac{d}{dt}\iint_G \rho F\, dx\, d\varphi - \int_g \rho(uF\, d\varphi - wh\, dx) = 0 \tag{1.6}$$

$$\frac{d}{dt}\iint_G \rho uF\, dx\, d\varphi - \int_g \{\rho u(uF\, d\varphi - wh\, dx) + pF\, d\varphi\} - \iint_G \{pF_x + (p^+ - p^-)\, yy_x\}\, dx\, d\varphi = 0$$

$$\frac{d}{dt}\iint_G \rho wyF\, dx\, d\varphi - \int_g \{\rho wy(uF\, d\varphi - wh\, dx) - pF\, dx\} - \iint_G \{pF_\varphi + (p^+ - p^-)\, yy_\varphi\}\, dx\, d\varphi = 0$$

$$\frac{d}{dt}\iint_G \rho(2e + q^2)F\, dx\, d\varphi - \int_g \rho(2i + q^2)(uF\, d\varphi - wh\, dx) = 0$$

where G is the projection of Ω on the plane of variables $x\varphi$, g is the boundary of G, $F = yh$, and $p^{\pm}$ are values of p on $\Sigma^{\pm}$, respectively. It is also assumed on the basis of the equation of state that e and, consequently, i are known functions of p and ρ. It should be noted that, although we deal with mean parameter values, the formulas that are valid for actual values are applicable here by virtue of (1.5).

In addition to the equations of state we need the expression for the difference $p^+ - p^-$ for closing the derived system. To obtain that expression (the condition of "radial equilibrium") we proceed as follows. Taking into account the constraint on the orientation of possible discontinuities defined by (1.5), we integrate the differential equation for the radial component of momentum over the duct length. Proceeding as previously and omitting the averaging symbol, we obtain

$$\frac{\partial \rho vF}{\partial t} + \frac{\partial \rho uvF}{\partial x} + \frac{\partial \rho wvh}{\partial \varphi} + (p^+ - p^-)\, y + \rho h(v^2 - w^2) = 0 \tag{1.7}$$

In conformity with (1.1), (1.2), and the condition of impermeability of $\Sigma^{\pm}$: $v \sim uy_x + wy_\varphi/y$, and, consequently, v^2 is almost everywhere smaller than u^2 and w^2, a feature that was already taken into account in the derivation of (1.6). By virtue of this and of the reasonable assumptions about the order of magnitude of derivatives in (1.7) we obtain the required condition

$$(p^+ - p^-)y = \rho w^2 h \tag{1.8}$$

which together with the equation of state closes (1.6).

Equations (1.6) and (1.8) admit on the one hand independent scales for x and y, and on the other for h. Hence, when conditions (1.1), which ensure the validity of the described approximation, are satisfied, we have similarity of flows with the same distributions of $h/h_0 = f^h(\varphi, x/X)$ and $y/X = f^y(\varphi, x/X)$, where f^h and f^y are functions of own arguments, and $X = x_1$ is the duct length. Henceforth the superscript zero (unity) will denote parameters in the inlet (outlet) cross section of the duct. This relation extends the similar property of flows in ducts that can be considered in a quasi-one-dimensional approximation, when only the distribution of relative areas of cross sections F/F_0 over x/X is important.

The conditions at discontinuities obtained in the conventional way from (1.6) coincide with those for actual parameters. Thus, if $D(t, x, \varphi) \equiv D^\circ(t, x, \varphi, y) = 0$ with $y(x, \varphi) \equiv (y^+ + y^-)/2$ is the equation of discontinuity and d is its velocity along the normal to itself (in terms of D_t, D_x, and D_φ), then on shock waves

$$[\rho(q_n - d)] = 0, \quad [p + \rho(q_n - d)^2] = 0, \quad [q_\tau] = 0, \quad [2i + (q_n - d)^2] = 0 \tag{1.9}$$

Here and in what follows $[\Phi] = \Phi_+ - \Phi_-$ is the difference of Φ on the two sides of the discontinuity, q_n and q_τ are components of $\mathbf{q}$, respectively, normal and tangent to the discontinuity, which in this case are determined in terms of u and w. At contact (tangential) discontinuities we similarly have

$$[p] = 0, \quad (q_n - d)_\pm = 0 \tag{1.10}$$

Finally, the differential equations which follow from (1.6) and (1.8) and are valid in the subregion of parameter continuity assume, after some transformations, the form

$$L_1 \equiv \frac{d\rho}{dt} + \rho\left(\frac{\partial u}{\partial x} + \frac{1}{y}\frac{\partial w}{\partial \varphi}\right) + \frac{\rho}{F}(uF_x + wh_\varphi) = 0, \quad L_2 \equiv \frac{du}{dt} + \frac{1}{\rho}\frac{\partial p}{\partial x} - \frac{w^2}{y}y_x = 0, \quad L_3 \equiv \frac{d\gamma}{dt} + \frac{1}{\rho}\frac{\partial p}{\partial \varphi} = 0 \tag{1.11}$$

$$L_4 \equiv \frac{ds}{dt} \equiv \frac{1}{T}\left(\frac{di}{dt} - \frac{1}{\rho}\frac{dp}{dt}\right) = 0 \quad \left(\frac{d}{dt} = \frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + \frac{w}{y}\frac{\partial}{\partial \varphi}\right)$$

where d/dt is the operator of total differentiation with respect to time along the particle trajectory (for averaged stream), $\gamma = yw$, and T is the absolute temperature. The second form of equation $L_4 = 0$ is obtained by using the thermodynamic relation between the differentials of s, i, and p. Incidentally, that equation which is of a characteristic form ensures the constancy of entropy in a particle and in subregions of continuous flow.

2. When the flow is steady, the first term of operator d/dt vanishes, and it is then expedient to rewrite system (1.11) in the form

$$L_1^\circ \equiv \frac{q^2}{\rho u}\left(L_1 - \frac{T}{i_\rho}L_4\right) \equiv \frac{M^2-1}{\rho}\left(\frac{\partial p}{\partial x} + \frac{\zeta}{y}\frac{\partial p}{\partial \varphi}\right) + \frac{u^2}{y}\left(\frac{\partial \zeta}{\partial \varphi} - y\zeta\frac{\partial \zeta}{\partial x}\right) + \frac{q^2}{F}(F_x + \zeta h_\varphi) = 0 \tag{2.1}$$

$$L_2^\circ \equiv wL_1 - uL_2 \equiv \frac{1}{\rho}\left(\zeta\frac{\partial p}{\partial x} - \frac{1}{y}\frac{\partial p}{\partial \varphi}\right) - u^2\left(\frac{\partial \zeta}{\partial x} + \frac{\zeta}{y}\frac{\partial \zeta}{\partial \varphi}\right) -$$

$$\frac{\zeta q^2}{y}y_x = 0, \quad L_3^\circ \equiv L_2 + \frac{\zeta}{y}L_3 + \frac{T}{u}L_4 \equiv \frac{\partial I}{\partial x} + \frac{\zeta}{y}\frac{\partial I}{\partial \varphi} = 0, \quad L_4^\circ \equiv \frac{1}{u}L_4 \equiv \frac{\partial s}{\partial x} + \frac{\zeta}{y}\frac{\partial s}{\partial \varphi} = 0$$

where $M = q/a$ is the Mach number, $a = \sqrt{\rho i_\rho / (1 - \rho i_p)}$ is the speed of sound, $i_\rho = (\partial i / \partial \rho)_p$, $i_p = (\partial i / \partial p)_\rho$, and $\zeta = w/u$ defines the direction of the "two-dimensional" vector $\mathbf{q}$, and $I = i + q^2/2$ is the total enthalpy. In (2.1) not only the last equation but, also, the last but one are of the characteristic form that ensures the constancy of I on a streamline ($d\varphi / dx = \zeta / y$ is the equation of a streamline). The type of subsystem of the first two of Eqs. (2.1) depend on M. When $M < 1$ the system is elliptic and when $M > 1$ it is hyperbolic. In the second case every point of the "flow plane" $x\varphi$ is traversed besides a streamline (c^0 characteristic) by two more characteristics (the c^+ and c^- characteristics, or the first and second set characteristics). As implied by (2.1), the latter are, as in the plane and axisymmetric cases, at the Mach angle $\alpha = \arcsin(1/M)$ to the streamlines. If $\beta = \operatorname{ctg}\alpha = \sqrt{M^2-1}$, the last statement with the so-called compatibility condition is expressed by equations of the form

$$\frac{D^\pm\varphi}{Dx} = \frac{\beta\zeta \pm 1}{y(\beta \mp \zeta)}, \quad \frac{D^\pm\zeta}{Dx} \pm \frac{\beta}{\rho u^2}\frac{D^\pm p}{Dx} + \frac{\beta\zeta(1+\zeta^2)}{(\beta \mp \zeta)y}y_x \mp \frac{1+\zeta^2}{(\beta \mp \zeta)F}(F_x + \zeta h_\varphi) = 0 \tag{2.2}$$

where $D^\pm/Dx$ are operators of total differentiation with respect to x along a characteristic, and the upper (lower) signs correspond to the c^+- (c^--) characteristic. The first of equalities (2.2) defines the direction of characteristics; the second represents compatibility conditions which differ from the respective conditions for the axisymmetric flow by the substitution of w for v and in the form of free terms.

3. When the gas flows in an axisymmetric annular duct and the lack of axial symmetry in the stream is due only to small unsteady perturbations, it is expedient to use variational equations obtained by the linearization of (1.11). We denote parameter variations by $u^\circ, w^\circ, \ldots$ and their steady values by $U, W, \ldots$. The latter satisfy the system obtained in this case from (1.11) and (2.1) by rejecting derivatives with respect t and φ. The obtained system is integrable in quadratures and yields the following fairly obvious relations:

$$RUF = \text{const}, \quad 2I + U^2 + W^2 = \text{const}, \quad \Gamma \equiv yW = \text{const}, \quad S = \text{const} \tag{3.1}$$

where, unlike in Sect. 2, I is not the total enthalpy but the fixed value of specific enthalpy. For a duct of specified shape, i.e. for given $F = F(x)$ and $y = y(x)$, formulas (3.1) together with the equations of state and conditions at the in- and outlet of the duct, which determine the constants, make it possible to determine the distribution of all steady state parameters of the stream with respect to x. We define $u^\circ, \ldots$ by the equalities

$$u = U(1 + u^\circ), \; p = P(1 + p^\circ), \; \rho = R(1 + \rho^\circ), \; w = W + w^\circ \tag{3.2}$$

substitute (3.2) into (1.11), and carry out linearization. Restricting further analysis to perfect gas with the adiabatic exponent $\varkappa$, and omitting henceforth the superscript "o", denote by r, l, γ and s the following combinations of perturbations:

$$2r = u + \frac{p}{\varkappa M}, \quad 2l = u - \frac{p}{\varkappa M}, \quad \gamma = yw, \quad s = p - \varkappa\rho \tag{3.3}$$

in which $M = U / A$ is the Mach number determined by the steady x-component of velocity U and the speed of sound A. We call the combinations (3.3) perturbations of the right and left Riemann invariants of circulation and entropy. The equations which they satisfy can be represented as

$$\frac{D^+r}{Dt} = a_{11}r + a_{12}l + a_{13}\gamma + a_{14}s - \frac{A}{2y^2}\frac{\partial\gamma}{\partial\varphi}, \quad \frac{D^-l}{Dt} = a_{21}r + a_{22}l + a_{23}\gamma + a_{24}s + \frac{A}{2y^2}\frac{\partial\gamma}{\partial\varphi} \tag{3.4}$$

$$\frac{D\gamma}{Dt} = AU\frac{\partial}{\partial\varphi}(l-r), \quad \frac{Ds}{Dt} = 0, \quad \frac{D}{Dt} = \frac{\partial}{\partial t} + U\frac{\partial}{\partial x} + \frac{W}{y}\frac{\partial}{\partial\varphi}, \quad \frac{D^\pm}{Dt} = \frac{D}{Dt} \pm A\frac{\partial}{\partial x}$$

$$a_{11} = \frac{A}{2(M^2-1)}\left\{\left[\frac{1-\varkappa}{2}(M-1)M^2 - 1 - 3M\right]\sum + \left[\frac{1-\varkappa}{2}(M-1) - 1 - 3M\right]\chi\right\}, \quad \Sigma = \frac{d\ln F}{dx}, \quad \chi = \frac{W^2}{A^2}\frac{d\ln y}{dx}$$

$$a_{12} = \frac{A}{2(M+1)}\left[\left(\frac{\varkappa-1}{2}M^2 - 1\right)\Sigma + \frac{\varkappa-3}{2}\chi\right], \quad a_{13} = a_{23} = \frac{W}{Uy}\frac{d\ln y}{dx}, \quad a_{14} = a_{24} = \frac{U(M^2\Sigma + \chi)}{2\varkappa(M^2-1)M^2}$$

$$a_{21} = \frac{M+1}{M-1}a_{12}, \quad a_{22} = \frac{A}{2(M^2-1)}\left\{\left[\frac{1-\varkappa}{2}(M+1)M^2 + 1 - 3M\right]\Sigma + \left[\frac{1-\varkappa}{2}(M+1) + 1 - 3M\right]\chi\right\}$$

In this case the possible discontinuity surfaces (compression shocks) are close to planes $x = \mathrm{const}$. Hence it is convenient to represent the equations of any of them in the form $x(t, \varphi) = X_s + \delta(t, \varphi)$, where X_s is the fixed coordinate of the shock, and $\delta(t, \varphi)$ is small in comparison with a characteristic dimension of the problem such as, for instance, the duct length. If the oncoming supersonic stream is free of perturbations ($x < X_s$) the linearization of (1.9) yields the following relations (refraction laws):

$$r_+ = \lambda_{rl}l_+ + \lambda_{r\delta}\Sigma\delta, \quad s_+ = \lambda_{sl}l_+ + \lambda_{s\delta}\Sigma\delta; \quad \delta_t + Wy^{-1}\delta_\varphi = \lambda_{\delta l}l_+ + \lambda_{\delta\delta}\Sigma\delta, \quad \gamma_+ = \lambda_{\gamma\delta}\delta_\varphi \tag{3.5}$$

$$\lambda_{rl} = \frac{(1-2M_+)M_-^2+1}{(1+2M_+)M_-^2+1}, \quad \lambda_{r\delta} = \frac{(1-K)N-E}{[(1+2M_+)M_-^2+1]M_+}, \quad \lambda_{sl} = \frac{\varkappa(1-\lambda_{rl})(1-N)}{M_+M_-^2}, \quad \lambda_{s\delta} = \frac{\varkappa[M_+(N-1)\lambda_{r\delta} - E]}{N}$$

$$\lambda_{\delta l} = \frac{1 - M_+ - (1+M_+)\lambda_{rl}}{(K-1)M_+}U_-, \quad \lambda_{\delta\delta} = \left[\frac{(1+M_+)\lambda_{r\delta}}{(1-K)M_+} - 1\right]U_-$$

$$\lambda_{\gamma\delta} = K - 1, \ K = U_- / U_+, \ N = M_-^2M_+^2, \ E = (\varkappa-1)(M_+^2 - M_-^2) / (\varkappa+1)$$

which must be satisfied for $x = X_s$. In these formulas $\delta_t = \partial\delta/\partial t$, $\delta_\varphi = \partial\delta/\partial\varphi$ and $\lambda_{rl}, \lambda_{r\delta}, \ldots$ have the meaning of coefficients of interaction.

4. Let us show how the developed here methods and equations can be used in the analysis of fairly complex unsteady flows. We begin by applying the theory derived in Sect.3 to the investigation of an unsteady stream in an annular duct of variable cross section area with supersonic flow at the inlet ($x = 0$), a closing shock inside the channel, and subsonic velocity at the outlet cross section ($x = 1$). As in the derivation of (3.5), we assume the absence of perturbations at the duct inlet, and that the unsteadiness is due to pressure perturbations that are nonuniform with respect to φ and finite with respect to t at $x=1$. The latter was specified by the formula $p_1(t, \varphi) = \varepsilon T(t)\Phi(\varphi)$, where ε is the oscillation amplitude, and functions T and Φ are nonzero, respectively in the intervals $0 < t < 0.5$ and $-\pi/2 < \varphi < \pi/2$. T and Φ are here of the form of isosceles triangles of unit height and the unit of time is the ratio of the duct length to the critical velocity of steady flow. The steady flow is potential, the Mach number at the inlet $M_0 = 1.4$, the mean oridinate y is a linear function of x with $y_0 \equiv y(0) = 0.5$, and $y_x = 0.1$. The distribution of F along x was specified by a third power polynomial with minimum and maximum ($F_1 / F_0 = 2$) at $x = 0$ and $x = 1$, respectively. The shock fixed coordinate $X_s = 0.3$. Since in the considered linear problem all perturbations are proportional to ε, they are replaced below by quantities obtained from (3.3)—(3.5) for $\varepsilon = 1$.

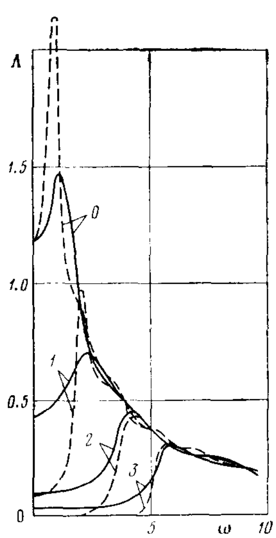


Fig.2

The unknown functions $r, l, \ldots$ are periodic in φ and in terms of Fourier integrals periodic in t. For r, δ, and p_1 we have

$$r(t, x, \varphi) = \sum_{n=-\infty}^{\infty} e^{in\varphi}\int_{-\infty}^{\infty} R(x, n, \omega)e^{i\omega t}d\omega \tag{4.1}$$

$$\delta(t, \varphi) = \sum_{n=-\infty}^{\infty} e^{in\varphi}\int_{-\infty}^{\infty}\Delta(n, \omega)e^{i\omega t}d\omega, \quad p_1(t, \varphi) \equiv T(t)\Phi(\varphi) = \sum_{n=-\infty}^{\infty} e^{in\varphi}\int_{-\infty}^{\infty} P(1, n, \omega)e^{i\omega t}d\omega$$

The equations for the determination of spectral densities $R, L, \Delta, \ldots$ are obtained from (3.4) and (3.5), and, in conformity with (3.3) and the last of equalities (4.1), the boundary condition at $x = 1$ is of the form

$$L_1 = R_1 - P_1 / \varkappa M_1$$

$$P_1 \equiv P(1, n, \omega) = \frac{1}{(2\pi)^2} \int_{-\pi}^{\pi} e^{-in\varphi} d\varphi \int_{-\infty}^{\infty} p_1(t, \varphi) e^{-i\omega t} dt = \frac{1}{(2\pi)^2} \int_{-\pi/2}^{\pi/2} \Phi(\varphi) e^{-in\varphi} d\varphi \int_{0}^{1/2} T(t) e^{-i\omega t} dt$$

where the second expression for F_1 takes into account the dependence of p_1 on t and φ selected above.

In the algorithm constructed by the authors spectral density was determined with the use of beforehand calculated transfer functions ("frequency characteristics") $\Lambda^\delta(n, \omega)$, $\Lambda^r(x, n, \omega), \ldots$ that correspond to $p_1(t, \varphi) = \exp i(n\varphi + \omega t)$. By virtue of the problem linearity $\Lambda^\delta, \Lambda^r, \ldots$ define all spectral densities in terms of P_1 by equalities of the type $R = \Lambda^r P_1$. Transfer functions were determined by a method close to that described in /1/. Without dwelling on some unimportant differences (such, as for instance, dependence on φ and, consequently, on n was not considered), we point out the following. Within fairly wide (from the point of view of accuracy of subsequent determination of $r, l, \ldots$) range of n and ω, their calculation on a computer using a FORTRAN program takes less than 10 min. The subsequent determination of flow for the chosen above $p_1(t, \varphi)$ took approximately 5 min. Some of the results of these calculations are shown in Figs. 2—4 by solid lines. All subsequent calculations are for $\varkappa = 1.4$.

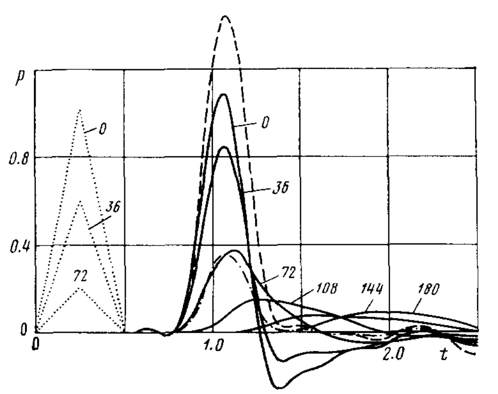


Fig.3

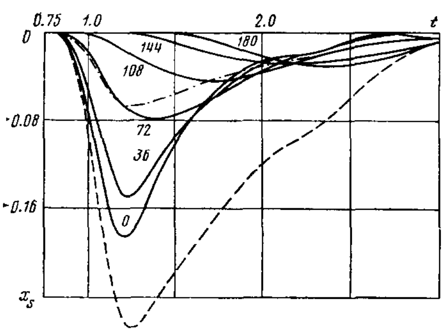
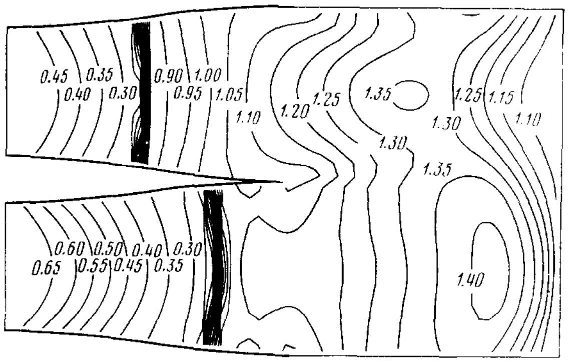


Fig.4

The dependence of $\Lambda = |\Lambda^\delta|$ on n and ω are shown in Fig.2, where the numbers at curves denote values of n; the frequency ω relates to the steady critical velocity divided by the duct length. It can be seen that Λ decreases as n is increased at considerable ω ; moreover Λ decreases (in some neighborhood of $\omega = 0$) also with decreasing ω. When $n \neq 0$ the nature of the latter effect is associated with that for $\omega < \omega_*(n)$, where ω_* is the so-called "critical" frequency or "cut-off" frequency /2,3/, solutions of the "travelling wave" type do not exist. It can be shown that in that case $\omega_* = An\sqrt{1 - M^2} / y$. The dash-line curves in Fig. 2 represent the similar relations for the original duct to which a cylindrical section of unit length has been added at its right-hand end. As expected, this virtually did not affect the right-hand (descending) branches of curves, diminished Λ for $\omega < \omega_*$, and increased the curvature of Λ growth sections lying near $\omega = \omega_*$, when $n \neq 0$. When $n = 0$, lengthening of the duct had increased The "resonance" peak at low ω.

Oscillaograms of pressure perturbation at the shock cross section are shown in Fig.3, where the numbers at curves indicate values of φ, and the dot-line triangles represent perturbing pressure pulses at the duct outlet. Oscillograms obtained by substituting some uniform distributions of $p_1 = p_1(t)$ for perturbations nonuniform with respect to φ. The dash line corresponds to $p_1 = T(t)\Phi(0)$ and illustrates the effect of lateral "creep" of perturbations. This effect is, however, not total, as shown by the dash-dot line which corresponds to $p_1 = T(t)\bar{\Phi}$. In this case $\bar{\Phi} = 0.25$ is the value obtained by averaging with respect to φ. Similar conclusions are implied by the oscillograms of variation of the shock coordinate (Fig.4).

Let us demonstrate now the possibilities provided by the numerical algorithm which, unlike the previous one, is based on the nonlinearized equations (1.6) and (1.8), and is intended for calculating unsteady flows with shock waves that are essentially nonuniform with respect to x and φ. An idea of one of such ducts is given in Fig.5 in which the "development" of the calculated region $d\eta = y d\varphi$ for $x = \text{const}$ in variables $x\eta$ is plotted, with isobars, and the



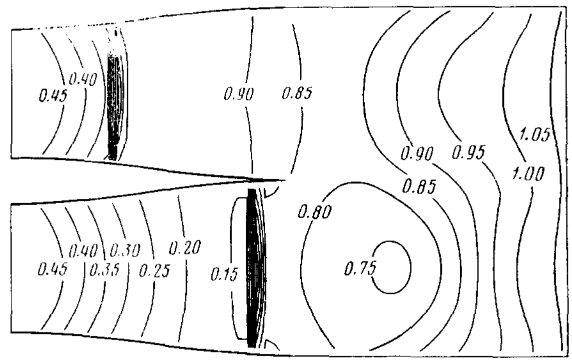


Fig.5

boundary g_w shown by a heavy line. The condition of impermeability $w / u = yd\varphi^g / dx$, where $\varphi = \varphi^g(x)$ is the equation of g_w, was imposed on g_w. Isobars were calculated at intervals $\Delta p = 0.05$. The numbers near some of them represent pressures normalized with respect to $\rho_* a_*^2$ which represent the critical density and velocity, respectively, of the steady stream at the duct inlet. The investigated configuration was obtained by the "truncation" of the axisymmetric annular duct which differed from the one described above only by the quantities $y_0 / X = 0.1$ and $y_x = 0.01$. It had two-sector inlets and a common annular outlet. Computations were carried out by Godunov's method /4/ without separation of discontinuity surfaces. The data presented below were obtained using a grid of 60 mesh on x and 30 on φ, and the FORTRAN computer program. Computation up to $t \sim 1.5$ (time normalized as previously) took approximately 1.5 hours of computer time. Isobars were constructed using a specially devised program for computing isolines. Initial steady parameter fields for a steady supersonic stream at the inlet (axial flow at $M_0 = 1.4$) and pressure $p_1 = 1.07$ constant with respect to φ at the outlet, were established in terms of t in the course of the computation process. As previously, closing shocks appeared in the duct. They are shown in Fig.5 by the isobars bunching zone.

After the steady parameter distribution was established, various perturbations were fed at the inlet or outlet, which imitated the arrival at the "lower" inlet branch (so called because of its position in Fig.5) of a shock wave or contact discontinuity and, also, of rotation in the impermeable outlet sector. In spite of fairly strong (perturbation) effects the flow, as a rule was not destroyed and remained stable. This agrees with the results of analysis in /3—8/.

The isobars shown in Fgi.5 relate to the problem of feeding to the lower duct branch of perturbations specified by formulas $\chi(t) = \chi_s\alpha(t) + \chi_0[1 - \alpha(t)]$, where χ is any parameter and χ_0 and χ_s are, respectively, its steady value and the value behind the shock wave which moves downstream at the relative velocity $1.2a_*$. For $0 < t < 1$ function $\alpha(t) = 1 - t$ and for $t > 1$ it vanishes. The time reference point was, as previously, set at the instant of the shock arrival to the cross section at $x = 0$. Pressure at $x = 1$ was maintained constant at $p_1 = 1.07$ throughout the process. At instant $t = 0.5$ (Fig.5,a) the closing shock in the upper branch has not yet "felt" the perturbation that has reached the lower branch. In Fig.5,b ($t = 1.3$) the stream is perturbed throughout the subsonic part of the duct, and the upper shock shifts to the left. However by then there are already no perturbations to the left of the lower shock. The data in Fig.5 show that the developed here mathematical model and algorithm make possible operational numerical experimentation with unsteady flows in ducts of fairly complex shapes.

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